

Fluid Mechanics - Course 223

REYNOLDS' NUMBER

When discussing viscosity we said that there were two basic types of flow - laminar and turbulent.

In a system with very low velocities the fluid may move in layers, one layer gliding over another, this is called LAMINAR flow. The effect of viscosity is to dampen any tendency towards instability of flow.

As the velocity increases, the viscosity is no longer able to control the instabilities and the flow starts to break up and become turbulent. This flow is called TRANSITIONAL flow. As the velocity increases further, the flow becomes fully TURBULENT.

A common example of the two types of flow is demonstrated by the rising column of smoke from a cigarette lying on an ashtray in a quiet room. For some distance the smoke rises in smooth filaments, which may wave around but do not lose their identity; this flow is LAMINAR. The filaments suddenly break up into a confused eddying motion at some distance above the cigarette; this flow is TURBULENT.

Turbulent flow occurs in the majority of fluid flow situations. Laminar flow occurs when there is low flowrate, low velocity and relatively high viscosity.

We may feel that this is an academic area with no practical application. We would be wrong. When evaluating the power that a pump has to provide, in a system, we need to know how much energy is being lost in the system. The energy loss depends upon many factors, one of which is whether the flow is laminar or turbulent. If the flow is laminar, the losses are closely proportional to the velocity of the fluid. If the flow is turbulent, the losses are roughly proportional to the square of the velocity.

We now have a problem. How do we know whether the system flow is fully laminar, transitional or fully turbulent? In an everyday situation we cannot examine the flow patterns.

Experiments by Reynolds to determine the type of flow, existing in a CIRCULAR pipe, demonstrated that there are many factors which affect the type of flow. As a result of his experiments he concluded that the type of flow depended upon the value of a dimensionless ratio, which is called

Reynolds' Number R_E .

$$\text{Thus } R_E = \frac{VD\ell}{\mu}$$

Where R_E = Reynolds' Number - dimensionless

D = pipe diameter - meters

V = average velocity - m/s

ℓ = density - kg/m^3

μ = dynamic viscosity N. sec/ m^2

Reynolds found that if R_E was less than 2,000, laminar flow existed and if R_E was above 4,000, turbulent flow was occurring.

From the above expression we can see that we have kinematic viscosity and R_E may be expressed in these terms:

$$R_E = \frac{VD}{\nu} \quad (\text{no units})$$

Examples

1. Oil of relative density 0.85, flows along a 6" SCH 40 pipe. The dynamic viscosity is 5×10^{-2} N.s/m.

Calculate (a) the maximum velocity with laminar flow.
 (b) the volumetric flowrate.
 (c) the mass flowrate.

R_E for maximum laminar velocity = 2,000

$$\text{Thus } 2,000 = \frac{VD\ell}{\mu}$$

$$\therefore V = \frac{2,000 \times \mu}{\ell \times D} \quad \text{m/s}$$

$$\text{Area of pipe} = \frac{\pi D^2}{4} = 186.4 \times 10^{-4} \text{ m}^2$$

$$\text{Thus } D = \left(\frac{186.4 \times 10^{-4}}{\pi} \right)^{1/2} = \underline{0.154 \text{ m}}$$

$$\ell = d \times 1,000 = 0.85 \times 1,000 = \underline{850 \text{ kg/m}^3}$$

$$V = \frac{2,000 \times 5 \times 10^{-2}}{850 \times 0.154} = \underline{0.764 \text{ m/s}}$$

$$Q_V = A \times V = 186.4 \times 10^{-4} \times 0.764$$

$$= 0.0142 \text{ m}^3/\text{s}$$

$$Q_M = \ell \times A \times V$$

$$= \ell \times Q_V$$

$$= 850 \times 0.0142$$

$$= \underline{12.1} \text{ kg/s}$$

2. If we had used oil of a lower viscosity, in the previous example, would the flow pattern change (Say $\mu = 1 \times 10^{-2}$).

$$R_E = \frac{VD\ell}{\mu}$$

$$= \frac{0.764 \times 0.154 \times 850}{1 \times 10^{-2}}$$

$$= \underline{10,000}$$

Thus the flow is now turbulent, although the density diameter and velocity are still the same.

ASSIGNMENT

1. How is the type of fluid flow affected by
 - (a) Velocity.
 - (b) Density.
 - (c) Pipe diameter.
 - (d) Dynamic Viscosity.
 - (e) Reynolds' Number.
2. C.C.W. water flows in a pipe at a rate of $1.5 \text{ m}^3/\text{s}$.
 $R_E = 10^6$, $\ell = 1,000 \text{ kg/m}^3$, $\mu = 7 \times 10^{-4} \text{ N.s/m}^2$.
 Calculate the diameter of the pipe (Answer 2.728 meters).
3. Considering the previous question, using the same flow-rate and liquid, how could laminar flow be achieved? Show your calculations for any changes you would make to the system.

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